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A Simple Method for Predicting the Capacity of a Lake for Development Based on Lake Trophic Status

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A general technique is presented for calculating the capacity of a lake for development based on quantifiable relationships between nutrient inputs and water quality parameters reflecting lake trophic status. Use of the technique for southern Ontario lakes is described. From the land use and geological formations prevalent in a lake's drainage basin, the phosphorus exported to the lake in runoff water can be calculated, which, when combined with the input directly to the lake's surface in precipitation and dry fallout, gives a measure of the natural total phosphorus load. From the population around the lake, the maximum artificial phosphorus load to the lake can be calculated and, if necessary, modified according to sewage disposal facilities used. The sum of the natural and artificial loads can be combined with a measure of the lake's morphometry expressed as the mean depth, the lake's water budget expressed as the lake's flushing rate, and the phosphorus retention coefficient of the lake, a parameter dependent on both the lake's morphometry and water budget, to predict springtime total phosphorus concentration in the lake. Long-term average runoff per unit of land area, precipitation, and lake evaporation data for Ontario provide a means of calculating the necessary water budget parameters without expensive and time-consuming field measurements. The predicted spring total phosphorus concentration can be used to predict the average chlorophyll *a* concentration in the lake in the summer, and this, in turn, can be used to estimate the Secchi disc transparency. Thus, the effects of an increase in development on a lake's water quality can be predicted. Conversely, by setting limits for the "permissible" summer average chlorophyll *a* concentration or Secchi disc transparency, the "permissible" total phosphorus concentration at spring overturn can be calculated. This can be translated into "permissible" artificial load, which can then be expressed as total allowable development. This figure can be compared to the current quantity of development and recommendations made concerning the desirability of further development on the lake.

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Les auteurs décrivent une méthode générale permettant de calculer la capacité de développement d'un lac. Cette méthode est fondée sur les relations quantifiables entre l'apport d'éléments nutritifs et les paramètres de qualité de l'eau reflétant la statut trophique de lac. Ils expliquent comment la méthode peut être appliquée à des lacs du sud de l'Ontario. A partir de l'utilisation des terres et des formations géologiques qui prévalent dans le bassin hydrographique d'un lac, on peut calculer le phosphore qui pénètre dans le lac avec l'eau de ruissellement, lequel, combiné à l'apport qui tombe directement à la surface du lac par précipitation ou retombée sèche, donne une mesure de la charge naturelle totale de phosphore. En se basant sur la population qui vit autour du lac, on peut calculer la charge artificielle maximale que peut recevoir le lac et, si nécessaire, la modifier en fonction des systèmes d'égoûts existants. La somme des charges naturelles et artificielles peut être combinée à la morphométrie du lac, exprimée en profondeur moyenne, au budget hydrologique du lac, exprimé en terme de vitesse de vidange du lac, et au coefficient de rétention du phosphore du lac, paramètre dépendant à la fois de la morphométrie et du budget hydrologique du lac, pour prédire la concentration printannière totale de phosphore dans le lac. Le ruissellement moyen à long terme, par unité de superficie de terre, la précipitation et l'évaporation dans l'Ontario sont des données qui permettent de calculer les paramètres de budget hydrologique sans avoir recours à des mesures longues et dispendieuses sur le terrain. La concentration printannière totale prédite de phosphore peut servir à prédir la concentration moyenne de chlorophylle *a* du lac en été et celle-ci, à son tour, peut être utilisée pour estimer la transparence au disque de Secchi. On peut, de cette façon, prédire les effets d'un développement accru sur la qualité de l'eau du lac. Inversement, en établissant des limites à la concentration estivale moyenne de la chlorophylle *a* ou de la transparence au disque de Secchi « permissibles », on peut calculer la concentration totale « permissible » de phosphore au moment du brassage général de printemps. Ceci peut être traduit en terme de charge artificielle « permissible », pour enfin être exprimé sous forme de développement total permissible. Ce chiffre peut être comparé au degré de développement présent et servir de base à des recommandations quant aux avantages de développements plus poussés sur le lac.

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THE demand for summer cottages and other recreational facilities in Ontario is increasing rapidly, with about 10,000 new cottages under construction in each year (Department of Tourism and Information 1971). Governmental planners, biologists, and engineers must evaluate and subsequently approve or reject proposed developments without guidelines that quantitatively determine the environmental impact of such development. Suitable guidelines must be based on predictive ecological and social theories that, to date, have not been formulated in terms of practical management tools. It is the aim of this paper to provide a means of determining the capacity of a lake for cottage or permanent home development, assuming that specific acceptable limits can be established for some parameters that reflect the quality of the aquatic environment. Translation of these limits to permissible cottage numbers or other development schemes can be achieved through application of techniques described in this report. Conversely, if a decision is made to allow the development of a given number of cottages or large-scale condominiums, then the governmental agency responsible will be completely cognizant of the environmental ramifications of its decision.

At this time, only the problem of lake trophic status is considered, with the equally important concerns of the effects of development on fisheries, wildlife, and on human health not taken into account (there is, of course considerable relationship between trophic state and fisheries potential, as well as health hazards). It should be stressed that the approach described herein is limited by its inability to consider these alternate parameters.

In understanding lake conditions, it is important to realize that the entire watershed and not just the lake or the lake and its shoreline, is the basic ecosystem unit. The terrestrial and aquatic portions of any watershed are inherently linked with the gravitational movements of minerals in drainage waters flowing from the land to the water (i.e. lakes, oceans), as the major terrestrial–aquatic linkage (Likens and Bormann 1974). It is important to note that the aquatic portion of a watershed is "downhill" from the terrestrial portion; that is, dissolved and particulate materials from the land are transported by geological process to the water and eventually to the less accessible sites of the sediments in the aquatic portions of the ecosystem. There is, of course, some "uphill" movement of material by meteorological and biological transport, e.g. release of N_2

from a lake through denitrification and gas diffusion, followed by precipitation of the same to the land for the former process and movement of material through the food chain for the latter. However, the net movement is always "downhill" to the aquatic system, with the deficit made up by geological weathering of rocks. The implication of this in terms of a lake's capacity for development is that any alteration in a watershed ultimately affects the lake. Thus, any management approach based solely on the lake and its shoreline is both simplistic and inaccurate.

Although a number of attempts to manage lake fisheries by the empirical approach have been tried (Moyle 1946; Rawson 1952, 1955; Ryder 1965), very little has been accomplished in terms of managing lakes for development with regard to trophic status by a similar method. An exception was Seppanen's proposal (1972) to determine a lake's summer cottaging capacity. The author suggested that a suitable formula for the recommended number of shoreline cottages was

$$\frac{A\sqrt{D_L}}{10}$$

where A is the lake area in hectares, D_L is the development of shoreline equal to the shorelength divided by the circumference of a circle of the same area, and 10 is a figure representing the minimum area of lake surface (in hectares) needed per cottage for a round lake with no islands. While providing a formula that takes into account both lake area and shoreline length, many other important factors are neglected limiting the technique. An entirely different, although somewhat indirect, approach has recently been used in a lake management sense. In 1968, Vollenweider published a plot of total annual phosphorus loading L (the amount of material added per unit surface area of lake per annum) vs. mean depth ($\bar{z}$). He found that bands could arbitrarily be drawn separating the lakes into the three standard lake types in terms of degree of eutrophy: oligotrophic, mesotrophic, and eutrophic. In 1973, Vollenweider modified his relationship to take hydraulic flushing rate into account. This was accomplished by incorporating the mean residence time of the water in the lake (τ_w). By plotting L vs. $\bar{z}/\tau_w$, a more realistic representation was achieved. The parameter $\bar{z}/\tau_w$ is equivalent to the areal water loading (i.e. the height of the water load over the lake's area that is supplied in 1 yr). The L vs. $\bar{z}/\tau_w$ model has been used for predicting changes in trophic status of several Muskoka Lakes (Michalski et al. 1973) that would result from changes brought about by the reduction of phosphorus in sewage effluent. As effected by Michalski et al. (1973), the model can be employed by environmental managers in the same manner as the L vs. $\bar{z}$ plot: the maximum additional loading to a lake that will not surpass the "permissible" loading can be calculated and interpreted in terms of cottage development. Although a significant improvement over the L vs. $\bar{z}$ plot, this method still lacks quantitative predictibility in terms of water quality parameters.

Basis for a New Approach

It has been demonstrated on numerous occasions (Schindler et al. 1971; Schindler 1974; Fuhs et al. 1972) that phosphorus is the nutrient most frequently controlling production and therefore trophic status in north temperate lakes. Therefore, any approach to predict water quality from a trophic status point of view must take into account the importance of phosphorus. As early as 1947, Sawyer (1947) recognized that phosphorus concentration in the lake water was the factor controlling eutrophication. Although Vollenweider's L vs. $\bar{z}$ plot relates loading rather than concentration to trophic state, his improved second relationship, L vs. $\bar{z}/\tau_w$, relates phosphorus concentration, and not loading, to trophic state. This may seem at first to be a contradiction but the rationale is as follows: the lines separating lakes into distinct trophic types have the dimensions of $L/\bar{z}/\tau_w$ or g m^{-2} yr^{-1}/m yr^{-1}, i.e. g m^{-3}. Thus, the lines are independent of time and have units of concentration (the lines of the L vs. $\bar{z}$ plot have units of g m^{-3} yr^{-1}, i.e. of volumetric rather than areal loading). Since the parameters with which one subjectively evaluates a lake (chlorophyll *a*, oxygen in the hypolimnion) are also expressed in terms of concentration, the above interpretation is most reasonable. Consequently, predictions relating the impact of development on the phosphorus concentration of a lake and subsequently on parameters describing the trophic state are central to a predictive management scheme.

The overall approach employed here is shown in Fig. 1. From consideration of the geology and land use of a lake's drainage basin, it is possible to estimate the total phosphorus exported or washed out per unit of watershed, which, combined with the drainage area, provides an estimate of the total phosphorus supplied to the lake from the land. Addition of the input of phosphorus in precipitation directly falling on the lake allows calculation of the natural phosphorus load to the lake. Development existing on the lake is then measured (aerial survey or field counts) and

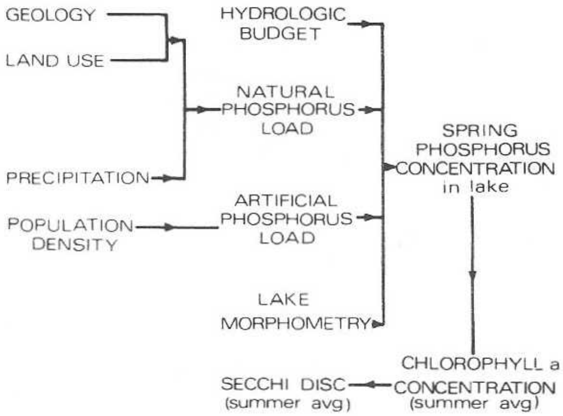

FIG. 1. Scheme of simple empirical models used to assess effects of development on trophic status of lakes.

the phosphorus loading from artificial sources calculated. The total loading, natural plus artificial, may then be combined with the lake's morphometry and water budget to predict a phosphorus concentration that is subsequently related to the average summer chlorophyll *a* concentration. From this latter calculation, one may determine the mean Secchi disc visibility. This method will most often be used in the opposite sense. For example, maximum acceptable average summer chlorophyll *a* concentration (or minimum Secchi disc reading) will be established by the appropriate governmental agency. From these limits, decision-making personnel will be able to calculate a maximum permissible phosphorus concentration, which can be interpreted in terms of a maximum "permissible" total phosphorus load, because the lake morphometry and water budget are essentially fixed. That is, although there are year-to-year variations in the lake's water budget, the long-term average budget is used. Finally, the maximum permissible artificial phosphorus loading can be estimated and expressed as the maximum allowable development (i.e. numbers of cottages, etc.). Although hypolimnetic oxygen deficits have been related to phosphorus loading for the Great Lakes (Gilbertson et al. 1972), a direct relationship applicable to a wide variety of lakes and especially to lakes of the Precambrian Shield is not yet available. Mortimer (1941–42) suggested a limit of 0.25 mg O_2 cm^{-2} day^{-1} for oligotrophic lakes, but the quantitative link to nutrients is, as yet, unformulated.

In the following section the theory and method for each step of the scheme are described and subsequently a stepwise procedure for calculating the lake's capacity is outlined for use by planners, managers, etc. Although the research work that the specific models are based on was largely carried out in southern Ontario and the particular values suggested for different parameters apply to southern Ontario lakes, the overall approach is general enough that with the appropriate data available, a similar sequence of predictions could be made for lakes of other regions.

Theory for Each Step of the Management Scheme for Southern Ontario Lakes

CALCULATION OF THE PHOSPHORUS LOADING TO THE LAKE

1. *Natural phosphorus load from land* (L_e) — The geological formations of southern Ontario can, as a first approximation, be classified as Precambrian igneous rock of plutonic origin (Canadian Shield) or sedimentary rock. The former is typically composed of granites, gneisses, pegmatites, syemite, migmatites, diorite, gabbro, hornblendite, amphibolite, and pyroxenites, the latter of limestone, dolomite, shale and basal clastics. A very large proportion of Ontario's recreational lakes are situated on the Shield, with only the southern portion of the Kawartha–Trent system, Lake Simcoe, and a few smaller lakes on sedimentary material. In addition, the watersheds of most of the Shield lakes are entirely or almost entirely forested, with the remainder being either marshland or pastureland that, although used for agricultural purposes, is not chemically fertilized. Even south of the Shield on the sedimentary bedrock, there are few areas where intensive agriculture (i.e. chemical fertilization) is practiced in prime recreational land; some areas around Lake Simcoe are a notable exception. Dillon and Kirchner (1975) have developed a phosphorus export scheme (Table 1) that is based on classification of geology as either "igneous" or "sedimentary" and land use as "forest" or "forest plus

TABLE 1. Ranges and mean values for export of total phosphorus (E) from 43 watersheds. Results in mg m^{-2} yr^{-1} (from Dillon and Kirchner 1975).

	Geological classification	
Land use	Igneous	Sedimentary
Forest		
Range	0.7–8.8	6.7–18.3
Mean	4.7	11.7
Forest + Pasture		
Range	5.9–16.0	11.1–37.0
Mean	10.2	23.3

pasture"; the latter category implies that 15% or more of the watershed is cleared but unfertilized land. This export scheme is applicable to almost the entire recreational lake area of Ontario. The results of Dillon and Kirchner are based on a combination of a study of the phosphorus export (E) of 34 watersheds in southern Ontario and all additional phosphorus export studies reported in the literature where watersheds fall into the above-mentioned categories. It is important to note that the natural export from sedimentary materials (off the Shield) is almost exactly double that from igneous bedrock (on the Shield); therefore, lakes such as the Kawarthas have a higher natural loading than Precambrian lakes. In addition, a change from a land use of forest to forest plus pasture doubles the export within a geological classification; for example, from 4.7 to 10.2 mg m^{-2} yr^{-1} for igneous watersheds and from 11.7 and 23.3 mg m^{-2} yr^{-1} in sedimentary watersheds. If a development significantly alters the amount of cleared land in a watershed, then an appropriate change in the export value used must be made.

It is also important to note that a range of values was measured for each category. Forested igneous watersheds, for example, exported 2.5–7.7 mg P m^{-2} yr^{-1} in the southern Ontario region alone, while Schindler et al. (1974) measured a range of 3–7 mg m^{-2} yr^{-1} in the Experimental Lakes Area. The total nutrient budget can obviously be significantly affected by the choice of export figure.

To calculate the natural phosphorus loading to a lake from its drainage area, one must know the area of the watershed (A_d) of each tributary to the lake, and be able to classify each as to geology and land use so that the export coefficient can be determined. Should the location be one of the few where intensive farming is undertaken, the reader should consult Vollenweider (1968), Dillon and Kirchner (1975), and Loehr (1974). The total amount of phosphorus supplied to the lake from the land is therefore calculated as the sum of the area of each drainage basin times its phosphorus export coefficient:

$$J_E(\text{mg}) = \Sigma\,(A_d \cdot E_i) \text{ with } A_d \text{ in m}^2. \qquad (1)$$

The areal loading of phosphorus supplied to the lake from the land L_E, is equal to $(\Sigma\, A_d\, E_i)/A_o$ and is equivalent to the supply divided by the lake area (A_o).

A complicating factor arises if any tributary or watershed of the lake in question has an additional lake or lakes in its course. These lakes undoubtedly act as traps for phosphorus and other nutrients, decreasing the actual amount of material transported from the drainage area to the lake in question. A means of accounting for this is described in a later section.

2. *Natural phosphorus load from precipitation* (L_{PR}) — Phosphorus input via precipitation (wet and dry fallout) has been virtually ignored until recent years. Studies by Schindler and Nighswander (1970), Armstrong and Schindler (1971), Barica and Armstrong (1971), Dillon and Rigler (1974a), Lee and Kluesener (1971) and Shannon and Brezonik (1972) have demonstrated that, for many lakes, precipitation can be a major source of nutrients. From consideration of the above-mentioned studies, which found loadings ranging from 24 to 102 mg m^{-2} of lake surface yr^{-1}, a value of 75 mg m^{-2} yr^{-1} is recommended as applicable to southern Ontario lakes. Some of the lower values did not include a measure of phosphorus in dry fallout, while others were measured in more remote areas far from urban and agricultural centres (Schindler et al. 1974) and the highest value, 102 mg m^{-2} yr^{-1}, was measured in an area where much of the land was used for agriculture. Furthermore, the figure of 77 mg m^{-2} yr^{-1} found by Dillon (1974a) was measured in the Haliburton Highlands area of the province, an area of prime concern for recreational development, although one close to much of the province's agricultural and industrial development. Thus, a figure of 75 mg m^{-2} yr^{-1}, for southern Ontario, although reasonable may be in error by as much as 100%. The year to year variation alone was greater than 100% in certain areas (Schindler et al. 1974). Additional research on this topic is obviously needed. A review has been provided by Chapin and Uttormark (1973).

With L_{PR} = 75 mg m^{-2} yr^{-1}, the natural phosphorus supply to a lake (J_N) is given as:

$$J_N = J_E + J_{PR}$$
$$= \sum (A_d \cdot E_i) + 75 \cdot A_o \quad (\text{mg yr}^{-1}) \quad (2)$$

or the loading (L_N)

$$L_N = \frac{\sum (A_d \cdot E_i)}{A_o} + 75 \qquad (\text{mg m}^{-2}\,\text{yr}^{-1})$$

3. *Artificial phosphorus load* (L_A) — The calculation of the phosphorus supplied to a lake by the population in its drainage basin is a difficult task and must necessarily be based on a supply per capita-year figure with several assumptions inherent in this method. Although numerous figures are quoted in the literature (e.g. see Vollenweider 1968 or Vollenweider and Dillon 1974, summary) great care must be taken in selecting

the appropriate value. The following points must be remembered:

a) Values measured in studies such as that of Johnson and Owen in 1971 (1.5 kg capita^{-1} y^{-1}) are no longer applicable because of legislation to reduce the phosphorus content of laundry detergents. High phosphorus content dishwashing detergents, however, remain legal and a survey of the Muskoka Lakes found that 30% of the cottages in the area employed automatic dishwashers.

b) The waste disposal technique for most developments that are already established or will be established in the near future is the conventional septic tank–tile field system. The efficiency of this treatment as far as phosphorus removal is concerned is dependent on the type and depth of soil surrounding the tile bed and between the tile bed and the lake. In Precambrian areas typically having very shallow, coarse-textured sandy or muck soils there is no satisfactory evidence which indicates that phosphorus is retained in the soils. Therefore, it must be assumed that all phosphorus discharged to soils of a tile bed area eventually gains access to the lake. In sedimentary areas, septic tank–tile field systems located in sand, gravel, or muck areas are likely to be as ineffective as far as phosphorus retention is concerned as those systems located on the Shield. Lakes surrounded by silt, clay, or clay–loam soil, however, will be provided with some measure of protection. Representative values for the fraction of phosphorus retained in some soils are given in Table 2.

c) Additional sources of phosphorus, (e.g. fertilizer applied to lawns in cottage areas), are impossible to evaluate quantitatively without a lake by lake survey.

TABLE 2. The retention coefficients of total phosphorus for septic tile filter beds of different characteristics. Results are based on 4 yr of data (after Brandes *et al.* 1974).

Filter bed	R_s
1. 22 in. sand (D_{10} = 0.24 mm) 8 in. mixture 4% red mud, 96% sand	0.76
2. 30 in. sand (D_{10} = 0.30 mm)	0.34
3. 30 in. sand (D_{10} = 0.60 mm)	0.22
4. 30 in. sand (D_{10} = 0.24 mm)	0.48
5. 30 in. sand (D_{10} = 1.0 mm)	0.01
6. 30 in. sand (D_{10} = 2.5 mm)	0.04
7. 15 in. sand (D_{10} = 0.24 mm) 15 in. mixture 10% red mud, 90% sand	0.88
8. 15 in. sand (D_{10} = 0.24 mm) 15 in. mixture 50% limestone, 50% sand	0.73
9. 30 in. silty sand	0.63
10. 15 in. sand (D_{10} = 0.24 mm) 15 in. mixture 50% clay–silt, 50% sand	0.74

With the above factors in mind, a reasonable phosphorus supply per capita-year can be calculated. According to the data of Bucksteeg (described in Vollenweider 1968), the annual per capita amount of phosphorus uptake in food in Germany in 1960 was 0.55 kg.; a similar amount is assumed to be excreted. In addition, the yearly per capita phosphorus supplied as domestic sewage (excrement plus household wastes) in 13 studies in North America and Europe averaged 0.80 kg. year^{-1}, the studies having been carried out before use of high phosphate detergents was common. Thus, the two results are in basic agreement, and with food consumption in North America being greater than that in Europe and there being continued usage of high phosphate dishwashing detergents in North America, the figure of 0.80 kg phosphorus capita^{-1} year^{-1} is a good estimate of the phosphorus supplied from domestic sources.

For all lakes situated on the Precambrian Shield the artificial supply of phosphorus can therefore be calculated as:

$$J_A = 0.8\ (\text{kg capita}^{-1}\ \text{yr}^{-1}) \times N\ (\text{cottages}) \times T\ (\text{capita-yr cottage}^{-1}\ \text{yr}^{-1}) \quad (3)$$

where N = the number of cottages or dwelling unit equivalents

T = the average number of capita years spent at each cottage in each year.

The same formula can be used for areas south of the Shield that consist of sand, gravel, or muck soils. In cases where holding tanks are employed and all wastes are removed to a treatment plant outside the lake's watershed, the supply of phosphorus from such dwellings should not be included in the calculations. Recent investigations (Brandes et al. 1974) involving the use of a red mud (aluminum mill waste) fill around the tile bed have demonstrated a potential phosphorus removal mechanism. In cases where such modifications to conventional tile bed systems are used and known to be in working order, a per capita phosphorus supply figure of 0.10 kg is suggested (88% removal). A factor $(1 - R_s)$, can be included in the model to allow for alterations in the phosphorus removal capacity of the system, with some values for R_s given in Table 2. The number of cottages or permanent dwellings (N) situated on a lake can be obtained by aerial photography or by field count. The time (in capita-years) spent at each dwelling per year (T) can be calculated using data gathered by the Department of Tourism and Information (1971). For cottages in southern Ontario, 253 capita days per year (or 0.69 capita-years per year) are spent at each cottage (based on a survey figure of 4.3 people per cottage). For permanent dwellings, the appropriate value for T is 1570 capita-days per year (or 4.3 capita-years per year). An actual field survey for a particular lake would undoubtedly improve these figures; nevertheless, they will serve as adequate, generally applicable estimates.

The total supply to the lake can be calculated as the sum of the natural and artificial supplies;

$$J_T = J_N + J_A = \Sigma(A_d \cdot E_i) + L_{PR} \cdot A_o + 0.8\ N \cdot T\ (1 - R_s) \quad (4)$$

where R_s is the retention capacity of the disposal system and will be equal to 0 for most Shield areas. The loading is, of course, equal to the supply per unit of lake area,

$$L_T = J_T / A_o.$$

Prediction of the Phosphorus Concentration in a Lake

A review of the theory and derivation of nutrient budget models published to the end of 1972 has been prepared by Dillon (1974b). Additional work by R. A. Vollenweider (unpublished data), Imboden (1973, 1974), Lerman (1974), Fleming (1974), W. J. Snodgrass and C. R. O'Melia (unpublished data), and Dillon (1975) is also highly pertinent to the topic. The model used here derived from Vollenweider (1969) is employed because a) it is simply derived and can be used with a minimum of field measurements and b) it alone has been tested for lakes in southern Ontario and has been found to have good predictive capabilities.

Vollenweider assumed that the change in the concentration of phosphorus in a lake with time is equal to the supply added per unit volume minus the loss through sedimentation and the loss by outflow:

$$\frac{d[P]}{dt} = \frac{J}{V} - \sigma[P] - \rho[P] \quad (5)$$

where [P] represents the phosphorus concentration, J is the amount of phosphorus supplied per annum, V is the lake volume, σ is the sedimentation rate (yr^{-1}), and ρ is the flushing rate (yr^{-1}) which is equal to the total volume of water outflowing per year (Q) divided by the lake volume (V).

The solution to this differential equation describing the concentration at a time t is:

$$[P] = \frac{L}{z(\sigma + \rho)}\left[1 - \left(1 - \frac{\bar{z}(\sigma + \rho)[P]_o}{L}\right) e^{-(\sigma+\rho)t}\right] \quad (6)$$

where $[P]_o$ is the initial concentration. The steady-state solution to this differential equation is:

$$[P] = \frac{L}{\bar{z}(\sigma + \rho)} \quad (7)$$

Equation (7) can be used to predict the phosphorus concentration in a lake if the parameters required for this, the loading (L), mean depth ($\bar{z}$), sedimentation rate (σ), and flushing rate (ρ), can be predicted or easily measured. While the loading is calculated as described in the preceding section and the mean depth is measured or obtained from previous lake surveys, the flushing rate (ρ) is approximated from the total outflow volume per year (Q) and the lake volume (V). Since the method outlined here is intended for use without field work if possible, a method of calculation of Q is required. The long-term average annual areal runoff (r, in m yr^{-1}) has been mapped for Ontario south of the Shield by Coulson (1967) and for the entire Great Lakes Basin by Pentland (1968). Thus, virtually the entire area of recreational lakes in Ontario is mapped in terms of r, where r represents the differences between precipitation and evapotranspiration. The total long-term average inflow via surface runoff to a lake can therefore be calculated as $A_d \cdot r$. The total water balance can now be described by the equation:

$$Q = A_d \cdot r + A_o\ (Pr - Ev) \quad (8)$$

where $A_o \cdot Pr$ represents the direct input of water to the lake by precipitation and $-A_o \cdot Ev$ represents loss from the lakes by evaporation. The average value for Pr for a given area can be obtained from isohyet maps (e.g. Canada Land Inventory 1966) of Canada or for particular areas, e.g. Ontario (Brown et al. 1968). Maps of Ev (evaporation from lake surface) are given for Canada by Bruce and Weisman (1966). If A_d is large compared to A_o, then Q can be simply approximated as $A_d \cdot r$. In the few studies undertaken in Ontario (e.g. Schultz 1950), groundwater has made a negligible contribution to the overall budget. The flushing rate, ρ, is then calculated as $Q(m^3\ yr^{-1})/V(m^3)$.

It must be cautioned that water budgets predicted in this manner may be very inaccurate, especially for small lakes and small drainage areas. Thus, Schindler et al. (1974) found measured water budgets for Rawson Lake to provide 3–5 times more water to the lake than predicted from precipitation, evaporation, and evapotranspiration figures. Dillon (1974a), however, found that measured values were generally within 25% of those predicted using long-term runoff maps, although these watersheds were much larger and might be expected to integrate out localized hydrologic variations. Measurement of the water budget rather than estimation would undoubtedly provide much more accurate results, and should be undertaken where possible.

Measurement of σ, the sedimentation rate coefficient, for phosphorus in a lake is, at best, extremely difficult. Fortunately, an alternate parameter, the retention coefficient, R, can be shown to have a functional relationship to σ. R is much more easily measured than σ; (R is equal to the fraction of the loading that is not lost via the outflow) as well, the retention coefficient has been shown to be predictable (Kirchner and Dillon 1975). Equation (7) used to predict the steady-state phosphorus concentration was rewritten by Dillon and Rigler (1974a) as:

$$[P] = \frac{L(1 - R)}{\bar{z}\rho} \qquad (9)$$

Kirchner and Dillon (1975) determined by multiple regresion analysis that R was highly correlated with Q/A, the areal water loading, usually written as q_s. Their equation for prediction of R is:

$$R = 0.426 \exp(-0.271\, q_s) + 0.574 \exp \times (-0.00949\, q_s) \qquad (10)$$

This basic formulation is almost identical to that predicted on theoretical grounds (Snodgrass 1974).

In some cases, the lake in question will have one or more lakes upstream that are sufficiently large to retain a significant amount of the total phosphorus exported from their respective portion of the watershed. This is taken into account by calculating the supply to the upstream lake, the lake's retention coefficient and multiplying the supply by $(1 - R_A)$ to give the fraction transfer to Lake B.

The Concept of Response Time of a Lake

As pointed out earlier, change in the phosphorus loading to a lake results in a change in the phosphorus concentration and a subsequent change in water quality. The change is, of course, not instantaneous but rather is described in equation (6) and shown in Fig. 2. The time required for

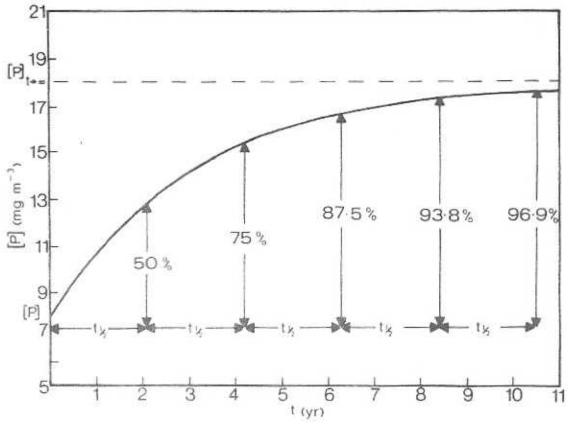

FIG. 2. Phosphorus concentration as a function of time for a lake with an increase in loading. The initial concentration was 8 mg m^{-3}, the final steady-state concentration is 18 mg m^{-3}, the half-life is 2.1 yr.

a lake having an initial loading of L_1 to respond to a change in loading to a new level L_2, (L_2 can be greater or less than L_1) can be described by the half-life of the change in concentration, $t_{\frac{1}{2}}$ (i.e. by the time required for the lake's phosphorus concentration to move half-way from the original steady-state concentration to the final steady-state concentration. Because equation (6) is exponential, two half-lives ($2t_{\frac{1}{2}}$) are required for the lake to reach 75% of its final concentration, $3t_{\frac{1}{2}}$ for 87½%, etc. The half-life, $t_{\frac{1}{2}}$, depends only on the rate coefficients σ and ρ, representing the losses of phosphorus by sedimentation and outflow; it is independent of the loading level or the initial phosphorus concentration in the lake. It can be easily shown that

$$t_{\frac{1}{2}} = \ln 2/(\sigma + \rho)$$

and using the approximation that $\sigma = 10/\bar{z}$ (R. A. Vollenweider unpublished data), the half-life can be estimated as:

$$t_{\frac{1}{2}} = 0.69/(\rho + 10/\bar{z}). \qquad (11)$$

Therefore, one must consider the response time of a lake when predicting the effects of an increased loading (e.g. resulting from development) or a decreased loading (e.g. from improved sewage treatment facilities). It is suggested that 3–5 times the $t_{\frac{1}{2}}$ (i.e. 87.5–96.9% of the way to the final steady-state concentration) can be used as an indicator of the lake's response time. Characteristically, lakes with a rapid flushing rate will have short half-lives and therefore response times, while lakes that are very slowly flushed may take a long time to respond to a change in loading. For example, a lake with flushing rate of $\rho = 1.0$ yr^{-1}

(i.e. the lake's volume is replaced once per year by flushing) and a mean depth of 10 m has a $t_{\frac{1}{2}}$ of $0.69/(1 + 10/10) = 0.35$ yr. Therefore, between 1.1 and 1.8 yr are required for the lake to approach a new steady-state following a change in loading.

Relationship of the Spring Phosphorus Concentration to the Summer Chlorophyll *a* Concentration and Water Transparency (Secchi Disc)

Based on the work of Sakamoto (1966), Dillon and Rigler (1974b) developed a predictive relationship suitable for estimating the average summer chlorophyll *a* concentration in lakes with spring N:P ratios >12, a conservative estimate of the lower limit for proportionate uptake. The data comprising this relationship were based on a combination of information presented by Sakamoto for 30 Japanese lakes, results for a wide variety of North American and European lakes reported by various authors, and information for 19 lakes in southern Ontario studied by Dillon and Rigler. The equation for the prediction of the summer average chlorophyll *a* concentration from the spring phosphorus concentration is:

$$\log_{10}[\text{chl } a] = 1.45 \log_{10}[\text{P}] - 1.14$$

with [chl *a*] and [P] in mg m^{-3}.

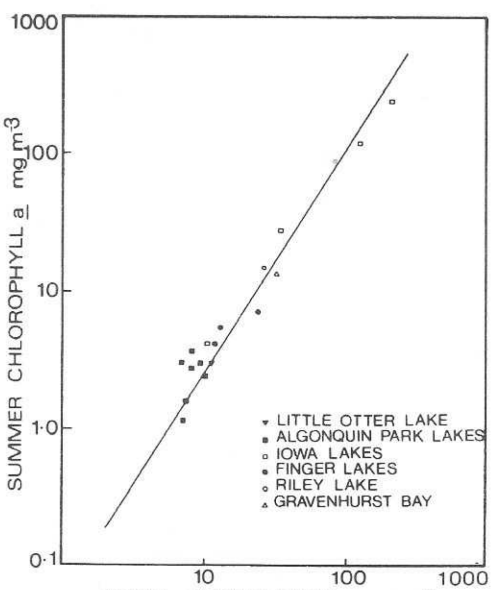

FIG. 3. Total phosphorus concentration at spring overturn vs. summer average chlorophyll *a* concentration for lakes not included by Dillon and Rigler (1974b).

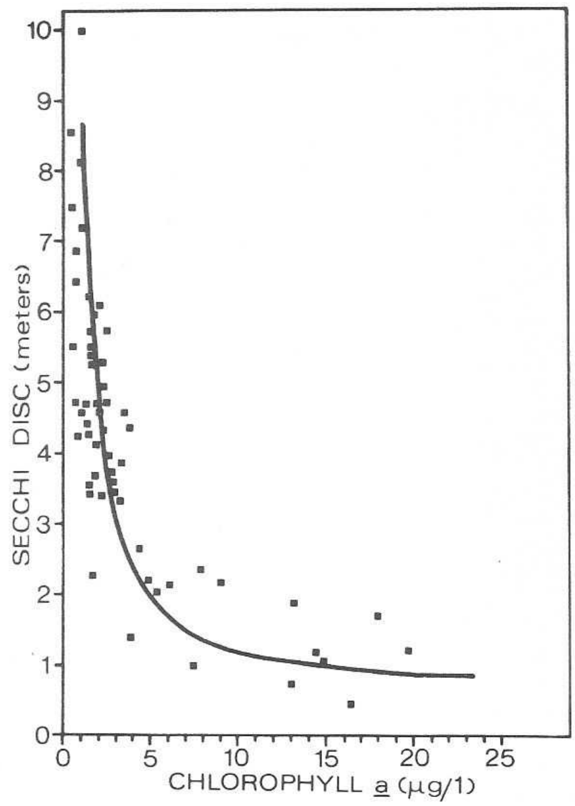

FIG. 4. The relationship between Secchi disc depth and chlorophyll *a* concentration for a number of lakes in southern Ontario. Values for each lake are based on means of values collected over the period of stratification (June–September).

Additional phosphorus–chlorophyll *a* data have been supplied by Scheider and Rigler (unpublished studies) for six lakes in Algonquin Park, by Oglesby and Shaffner for some of the Finger Lakes in New York State, and by Bachman and Jones (1974) for some Iowa lakes. These data are also shown in Fig. 3 and it is apparent that in all cases the fit to the model is very good.

It is expected that chlorophyll *a* concentration will be inversely related to the Secchi disc reading, a measure of the water's transparency. Collection of data for a large number of lakes is shown in Fig. 4. From the predicted chlorophyll *a* concentration, a prediction of the likely average Secchi disc reading is possible. Care must be taken in interpreting this relationship in the case of dystrophic (brown water) lakes, whose Secchi disc readings are lower than would be expected on the basis of chlorophyll *a* concentration alone.

Stepwise Procedure for Calculating the Development Capacity of a Southern Ontario Lake

Step 1. Based on long-range plans for the lake, decide what the maximum permissible average summer chlorophyll *a* concentration will be:

Level 1: 2 mg m^{-3}; for lakes to be used primarily for body contact water recreation, and where it is desirable to maintain hypolimnetic concentrations of oxygen in excess of 5 mg $liter^{-1}$ to preserve cold water fisheries. The lake will be extremely clear with a mean Secchi disc visibility of 5 m and will be very unproductive. (Note — the Secchi disc visibility may be lower in brown water [dystrophic] lakes).

Level 2: 5 mg m^{-3}; for lakes to be used for water recreation but where the preservation of cold water fisheries is not imperative. The lake will be moderately productive and correspondingly less clear, with a mean Secchi disc visibility of 2–5 m.

Level 3: 10 mg m^{-3}; for lakes where body-contact recreation is of little importance, but emphasis is placed on fisheries (bass, walleye, pickerel, pike, maskinonge, bluegill, yellow perch). Hypolimnetic oxygen depletion will be common. Secchi disc depths will be low (1–2 m), and there is a danger of winterkill of fish in shallow lakes.

Level 4: 25 mg m^{-3}; suitable only for warmwater fisheries. Secchi disc depth <1.5 m, hypolimnetic oxygen depletion beginning early in summer, considerable danger of winterkill of fish except in deep lakes. The planning agency may pick any intermediate level should it so desire.

Step 2: From the chosen summer average chlorophyll *a* concentration, calculate the permissible spring phosphorus concentration, [P] from:

$$\log_{10} [\text{chl } a] = 1.45 \log_{10} [P] - 1.14$$

$$\text{i.e. } \log_{10} [P] = \frac{\log_{10} [\text{chl } a] + 1.14}{1.45}$$

e.g.

[chl *a*]	[P]
2 mg m^{-3}	9.9 mg m^{-3}
5	18.5
10	29.9
25	56.3

Step 3: Determine the lake surface area (A_o in m^2), mean depth ($\bar{z}$ in m), and volume (V in m^3) from available information if possible. If such data are not available, the lake must be sounded and a contour map drawn. The area (A_o) is obtained by planimetry from an aerial photograph of known scale.

Step 4: Outline the lake's drainage area on a 1:50,000 scale topographic map or on aerial photographs and calculate the area (A_d in m^2) by planimetry.

Step 5: From Plate 13 in Pentland (1968) determine the total annual unit runoff (r) in cfs mi^{-2} and convert to $m^3\ yr^{-1}\ m^{-2}$ or $m\ yr^{-1}$ by multiplying by 0.345.

Step 6: If $A_d > 10\ A_o$ calculate Q, the total outflow volume as:

$$A_d \cdot r \ (m^3\ yr^{-1})$$

and calculate the flushing rate (ρ) as Q/V or

$$(A_d \cdot r)/V \ (yr^{-1})$$

If A_d is $<10\ A_o$, determine the mean annual precipitation (Pr) from figure 32 in Brown et al. (1968), and the mean annual lake evaporation (Ev) from figure 9 in Bruce and Weisman (1966), convert to m yr^{-1} by multiplying by 0.0254 and calculate Q:

$$Q = A_d \cdot r + A_o\ (Pr - Ev)$$

$$\therefore\ \rho = (A_d \cdot r + A_o\ (Pr - Ev))/V \qquad (yr^{-1})$$

Step 7: Calculate the areal water load as (q_s) as

$$Q/A_o \qquad (m\ yr^{-1})$$

Step 8: Calculate retention coefficient (R) as

$$R = 0.426 \exp(-0.271\ q_s) + 0.574 \exp(-0.00949\ q_s)$$

Step 9: Calculate the response time of the lake to a change in phosphorus loading:

$$\text{Response time} = (3 \rightarrow 5)\ t_{\frac{1}{2}}$$
$$= (3 \rightarrow 5)\ 0.69/(\rho + 10/\bar{z}) \qquad (yr)$$

This will provide an indication of the time required for a lake to "respond" to development and will give an idea of when follow-up studies (if any) should be carried out. Conversely, the response time can assist in the interpretation of present water quality. For example, a lake with 250 new cottages may appear to be in good condition, but if one can calculate that it has a response time of 6–10 yr, then caution is necessary before additional development is allowed.

Step 10: Calculate the permissible phosphorus load (L_{perm}) to the lake:

$$L_{perm} = \frac{[P]\cdot\bar{z}\cdot\rho}{(1-R)} \quad (\text{mg m}^{-2}\text{ yr}^{-1})$$

and the permissible supply (J_{perm})

$$J_{perm} = \frac{L_{perm}\cdot A_o}{10^6} \qquad (\text{kg yr}^{-1})$$

Step 11: Divide the watershed of the lake (on the topographic map) into subunits for all inflows and determine the area for each one (A_{di}). From the most recent aerial photographs available determine if >15% of the area of each watershed is either cleared land or marsh. Determine whether the watershed (not just the lake) is on Precambrian igneous rock or on sedimentary rock, and estimate the export (E mg m^{-2} yr^{-1}) for each subwatershed from Table 1. Calculate the total supply of phosphorus from the land to the lake:

$$J_E = (\Sigma_i A_{di} \cdot E) \qquad (\text{mg yr}^{-1})$$

and the load

$$L_E = (\Sigma_i A_{di} \cdot E)/A_o \qquad (\text{mg m}^{-2}\text{ yr}^{-1})$$

Take into consideration an upstream lake by reducing the supply to the downstream lake from the watershed containing the upstream lake by multiplying by $(1 - R^1)$ where R^1 is the phosphorus retention coefficient of the upstream lake. R^1 is calculated as in Step 8, using q_s for the upstream lake. R^1 is calculated as in Step 8, using q_s for the upstream lake.

Step 12: The phosphorus load from precipitation, L_{PR}, is 75 mg m^{-2} yr^{-1}.

Calculate the supply from precipitation as:

$$J_{PR} = \frac{75\cdot A_o}{10^6} \qquad (\text{kg yr}^{-1})$$

Step 13: The natural supply and natural loading are:

$$J_N = J_E + J_{PR}$$
$$L_N = L_E + L_{PR}$$

If $J_N \geqslant J_{perm}$, i.e. if the natural supply is greater than the permissible supply, allow no (further) development.

Step 14: Determine the present number of cottages (N_C) and permanent dwellings (N_D) within 300 m (1000 ft) of the lake or any of the inflowing streams or rivers from recent aerial photographs or field surveys. For cottages, assume 253 capita-days per year (0.69 capita-years yr^{-1}). Assume 4.3 people per dwelling, and calculate N_{CY}, the number of capita-years yr^{-1} spent at the lake:

$$N_{CY} = 0.69 \times N_C + 4.3 \times N_D$$

i.e. one permanent unit = 6.2 cottage units.

Step 15: calculate the phosphorus supplied to the lake from the cottage units (artificial supply) as:

$$J_A = 0.8 \times N_{CY}(1 - R_s) \qquad (\text{kg yr}^{-1})$$

where $R_s = 0$ for conventional septic tank–tile field systems on the Precambrian Shield. If there is firm evidence that holding tanks are used for all household wastes and the systems are pumped and removed to a treatment plant outside of the watershed, neglect such cottage(s) in the calculations. If the septic tile filter beds are situated off of the Shield on soils that correspond to those of Table 2, use the appropriate values for R_s.

Step 16: Calculate the present total of phosphorus to the lake:

$$J_T = J_N + J_A \qquad (\text{kg yr}^{-1})$$

If $J_T \geqslant J_{perm}$, allow no further development.

Step 17: If $J_T < J_{perm}$, calculate the total permissible number of cottage units:

$$N_{perm} = \frac{J_{perm} - J_N}{0.69 \times 0.8\,(1 - R_s)}$$

Step 18: The additional number of cottage units permitted is:

$$N_{add} = N_{perm} - N_{CY}/0.69$$

Discussion

The methodology outlined in the previous section provides a simple means of calculating the capacity for development of southern Ontario lakes based on lake trophic status. A number of previously tested empirical models are linked in a logical sequence relating watershed characteristics and utilization to water quality. Although the specific figures used are applicable only to southern Ontario, the approach is general enough to be applied to any lake, provided suitable data are available for parameters such as the phosphorus export coefficient, E, the loading from precipitation, L_{PR}, the phosphorus soil retention coefficient, R_s, etc. The advantages of this method are that quantitative predictions of the effects of development on important water quality parameters, chlorophyll *a* concentration and Secchi disc depth, are possible, and that these predictions can be made with no field work, provided the lake's morphometry is available. The disadvantages are that other important considerations, e.g. location of fish spawning grounds and wildlife habitat, microbiological contamination etc., are not taken into account, and that, although the individual models or relationships have been tested and found to make reasonably accurate predictions, for lakes in southern Ontario, overall predictions provided by the modeling sequence must be considerably less accurate. The uncertainty in the phosphorus export figures and in the loading from precipitation alone could result in a 100% error in the calculation of the natural phosphorus budget, while factors such as the soil retention factor are still only approximations. Nevertheless, these predictions should be valuable to those involved in lake management because of both their simplicity and their quantitative nature, provided proper caution is taken in interpretation of the results.

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